OCR Maths M2

Topic Questions from Papers

# Statics

1
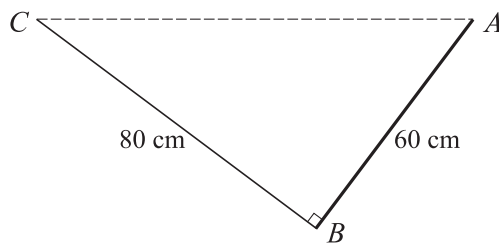
A uniform rod AB of length $60\,\text{cm}$ and weight $15\,\text{N}$ is freely suspended from its end A. The end B of the rod is attached to a light inextensible string of length $80\,\text{cm}$ whose other end is fixed to a point C which is at the same horizontal level as A. The rod is in equilibrium with the string at right angles to the rod (see diagram).

(i) Show that the tension in the string is $4.5\,\text{N}$. [4]

(ii) Find the magnitude and direction of the force acting on the rod at A. [6]

physicsandmathstutor.com

(Q5, June 2005)

2
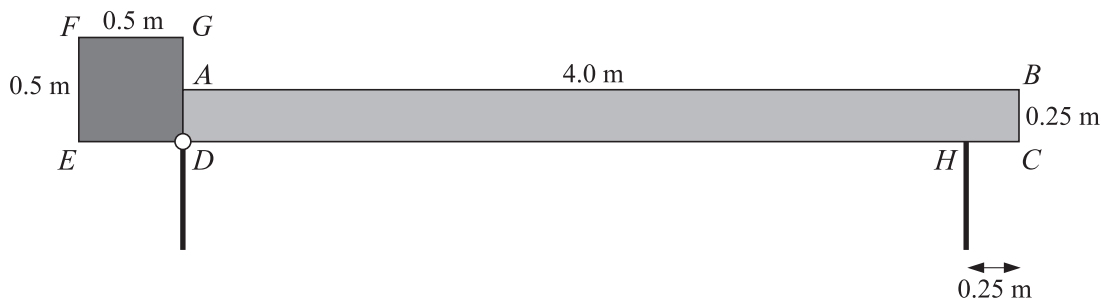
A barrier is modelled as a uniform rectangular plank of wood, $ABCD$, rigidly joined to a uniform square metal plate, $DEFG$. The plank of wood has mass $50\,\text{kg}$ and dimensions $4.0\,\text{m}$ by $0.25\,\text{m}$. The metal plate has mass $80\,\text{kg}$ and side $0.5\,\text{m}$. The plank and plate are joined in such a way that CDE is a straight line (see diagram). The barrier is smoothly pivoted at the point D. In the closed position, the barrier rests on a thin post at H. The distance CH is $0.25\,\text{m}$.
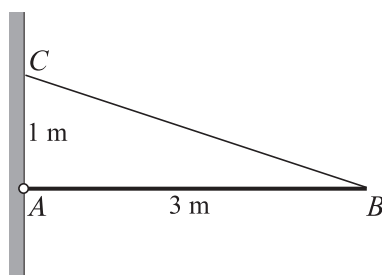
(i) Calculate the contact force at H when the barrier is in the closed position. [3]

In the open position, the centre of mass of the barrier is vertically above D.

(ii) Calculate the angle between AB and the horizontal when the barrier is in the open position. [8]

(Q7, June 2005)

3

A uniform rod AB has weight $20\,\text{N}$ and length $3\,\text{m}$. The end A is freely hinged to a point on a vertical wall. The rod is held horizontally and in equilibrium by a light inextensible string. One end of the string is attached to the rod at B. The other end of the string is attached to a point C, which is $1\,\text{m}$ directly above A (see diagram). Calculate the tension in the string. [4]

(Q1, Jan 2006)

4
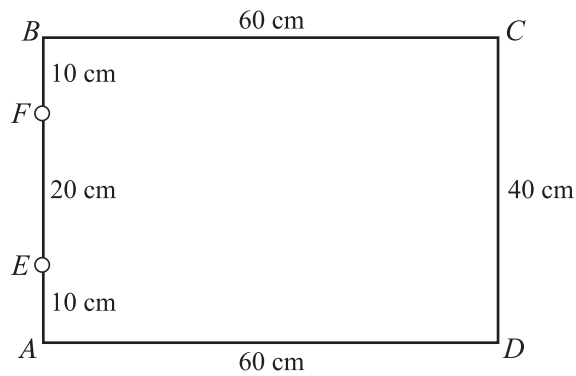
A rectangular frame consists of four uniform metal rods. *AB* and *CD* are vertical and each is 40 cm long and has mass 0.2 kg. *AD* and *BC* are horizontal and each is 60 cm long. *AD* has mass 0.7 kg and *BC* has mass 0.5 kg. The frame is freely hinged at *E* and *F*, where *E* is 10 cm above *A*, and *F* is 10 cm below *B* (see diagram).

(i) Sketch a diagram showing the directions of the horizontal components of the forces acting on the frame at *E* and *F*. [2]

(ii) Calculate the magnitude of the horizontal component of the force acting on the frame at *E*. [3]

physicsandmathstutor.com   *(Q4, Jan 2006)*
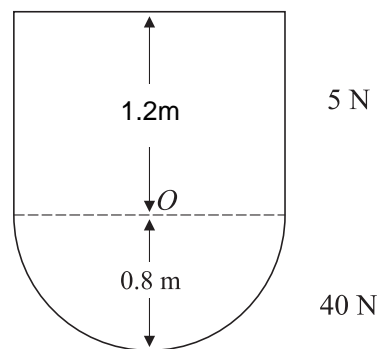
5

Fig. 1

An object consists of a uniform solid hemisphere of weight 40 N and a uniform solid cylinder of weight 5 N. The cylinder has height h m. The solids have the same base radius 0.8 m and are joined so that the hemisphere's plane face coincides with one of the cylinder's faces. The centre of the common face is the point *O* (see Fig. 1). The centre of mass of the object lies inside the hemisphere and is at a distance of 0.2 m from *O*.

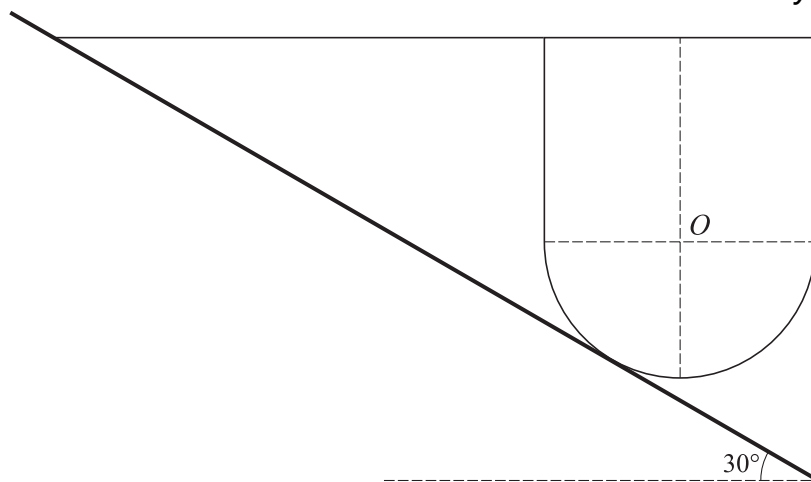*cont on next page*

physicsandmathstutor.com
Fig. 2

One end of a light inextensible string is attached to a point on the circumference of the upper face of the cylinder. The string is horizontal and its other end is tied to a fixed point on a rough plane. The object rests in equilibrium on the plane with its axis of symmetry vertical. The plane makes an angle of $30°$ with the horizontal (see Fig. 2). The tension in the string is T N and the frictional force acting on the object is F N.

(ii) By taking moments about O, express F in terms of T. [4]

(iii) Find another equation connecting T and F. Hence calculate the tension and the frictional force. [6]

(Q8, June 2007)

6
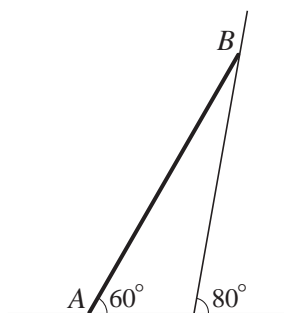
A uniform rod AB, of weight 25 N and length 1.6 m, rests in equilibrium in a vertical plane with the end A in contact with rough horizontal ground and the end B resting against a smooth wall which is inclined at $80°$ to the horizontal. The rod is inclined at $60°$ to the horizontal (see diagram). Calculate the magnitude of the force acting on the rod at B. [6]
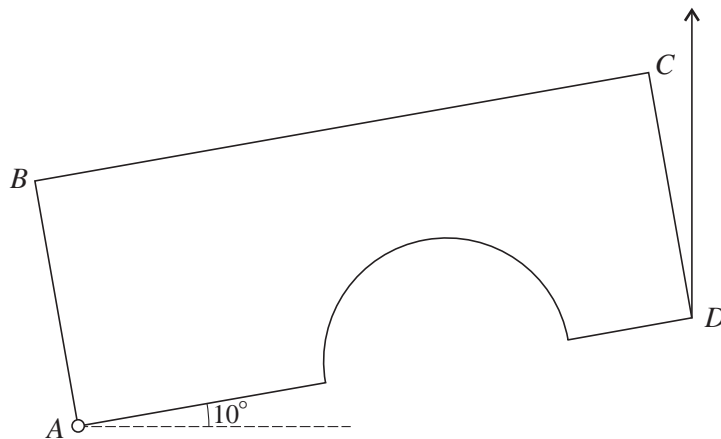
(Q3, Jan 2008)

7 (iii)

Fig. 2

The model bridge is smoothly pivoted at *A* and is supported in equilibrium by a vertical wire attached to *D*. The weight of the model is 15 N and *AD* makes an angle of 10° with the horizontal (see Fig. 2). Calculate the tension in the wire. [5]
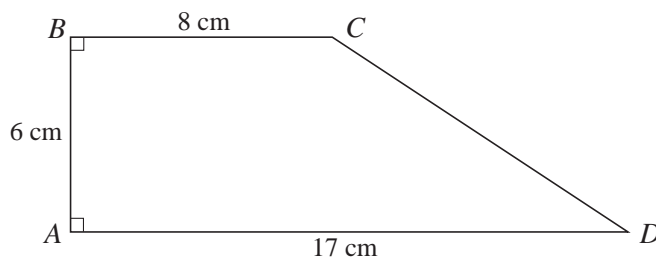
(Q8, Jan 2008)

8 (i)

Fig. 1

A uniform lamina *ABCD* is in the form of a right-angled trapezium. *AB* = 6 cm, *BC* = 8 cm and *AD* = 17 cm (see Fig. 1). Taking *x*- and *y*-axes along *AD* and *AB* respectively, find the coordinates of the centre of mass of the lamina. [8]
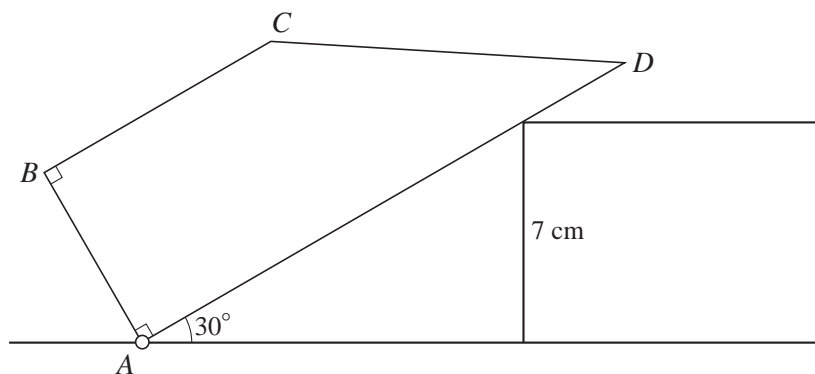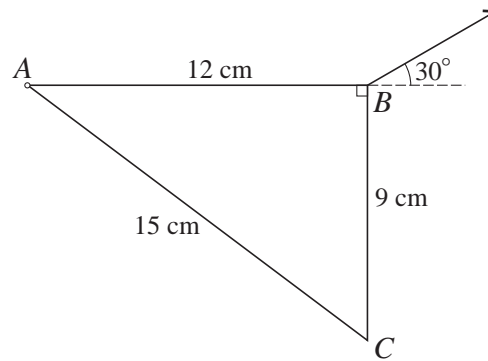
(ii)

Fig. 2

The lamina is smoothly pivoted at *A* and it rests in a vertical plane in equilibrium against a fixed smooth block of height 7 cm. The mass of the lamina is 3 kg. *AD* makes an angle of 30° with the horizontal (see Fig. 2). Calculate the magnitude of the force which the block exerts on the lamina. [5]
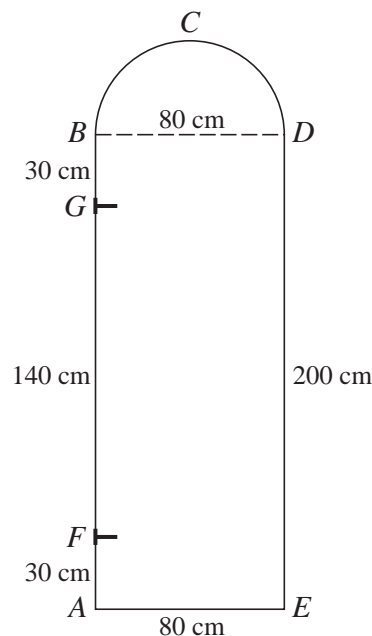
(Q8, June 2008)

9

A uniform right-angled triangular lamina ABC with sides $AB = 12$ cm, $BC = 9$ cm and $AC = 15$ cm is freely suspended from a hinge at its vertex A. The lamina has mass 2 kg and is held in equilibrium with AB horizontal by means of a string attached to B. The string is at an angle of $30°$ to the horizontal (see diagram). Calculate the tension in the string. **[4]**

(Q2, Jan 2009)

10

A door is modelled as a lamina $ABCDE$ consisting of a uniform rectangular section $ABDE$ of weight 60 N and a uniform semicircular section BCD of weight 10 N and radius 40 cm. AB is 200 cm and AE is 80 cm. The door is freely hinged at F and G, where G is 30 cm below B and F is 30 cm above A (see diagram).

  (i) Find the magnitudes and directions of the horizontal components of the forces on the door at each of F and G. **[4]**

(Q3, Jan 2009)

11
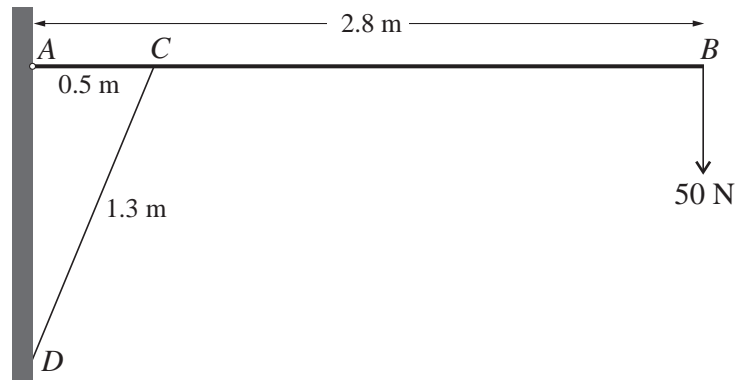
A uniform beam *AB* has weight 70 N and length 2.8 m. The beam is freely hinged to a wall at *A* and is supported in a horizontal position by a strut *CD* of length 1.3 m. One end of the strut is attached to the beam at *C*, 0.5 m from *A*, and the other end is attached to the wall at *D*, vertically below *A*. The strut exerts a force on the beam in the direction *DC*. The beam carries a load of weight 50 N at its end *B* (see diagram).

(i) Calculate the magnitude of the force exerted by the strut on the beam. **[4]**

(ii) Calculate the magnitude of the force acting on the beam at *A*. **[6]**
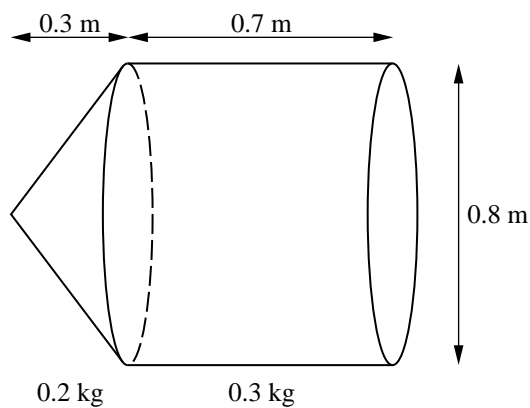
(Q3, June 2009)

12

Fig. 1

the distance of the centre of mass of the combined object from the vertex of the conical shell is 0.47 m.
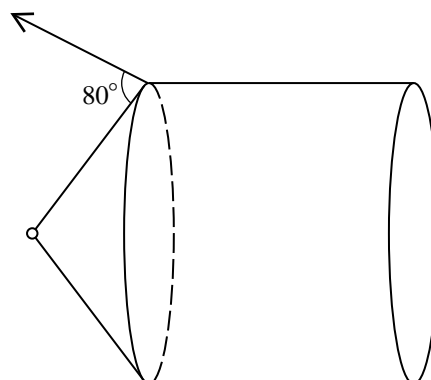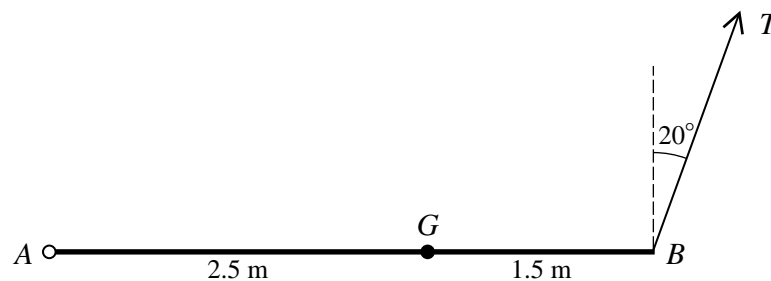

Fig. 2

The combined object is freely suspended from its vertex and is held with its axis horizontal. This is achieved by means of a wire attached to a point on the circumference of the base of the conical shell. The wire makes an angle of 80° with the slant edge of the conical shell (see Fig. 2).

(ii) Calculate the tension in the wire. **[4]**

(Q3, Jan 2010)

13

A non-uniform beam *AB* of length 4 m and mass 5 kg has its centre of mass at the point *G* of the beam where *AG* = 2.5 m. The beam is freely suspended from its end *A* and is held in a horizontal position by means of a wire attached to the end *B*. The wire makes an angle of 20° with the vertical and the tension is *T* N (see diagram).

(i) Calculate *T*. **[3]**

(ii) Calculate the magnitude and the direction of the force acting on the beam at *A*. **[7]**

(Q4, June 2010)

14 **(ii)**
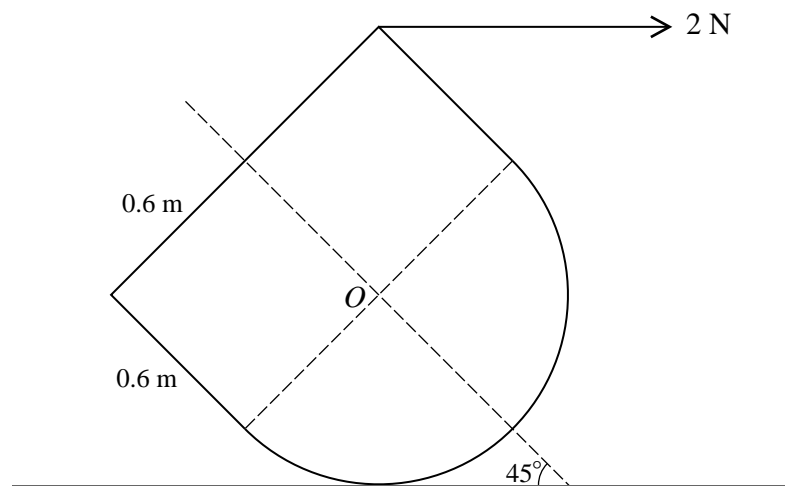
the distance of the centre of mass of the solid from *O* is 0.09 m.

The solid is placed with the curved surface of the hemisphere on a rough horizontal surface and the axis inclined at 45° to the horizontal. The equilibrium of the solid is maintained by a horizontal force of 2 N applied to the highest point on the circumference of its plane face (see diagram). Calculate

(a) the mass of the solid, **[4]**

(b) the set of possible values of the coefficient of friction between the surface and the solid.
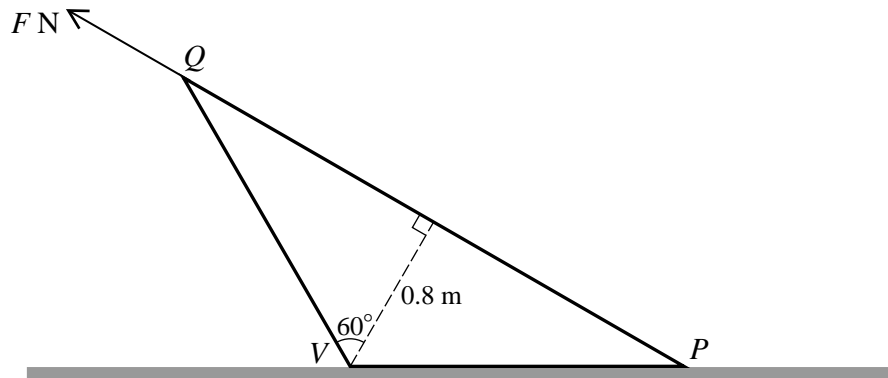
[3]

(Q5, Jan 2011)

15

Fig. 1

A uniform solid cone of height $0.8\,\text{m}$ and semi-vertical angle $60°$ lies with its curved surface on a horizontal plane. The point P on the circumference of the base is in contact with the plane. V is the vertex of the cone and PQ is a diameter of its base. The weight of the cone is $550\,\text{N}$. A force of magnitude F N and line of action PQ is applied to the base of the cone (see Fig. 1). The cone topples about V without sliding.

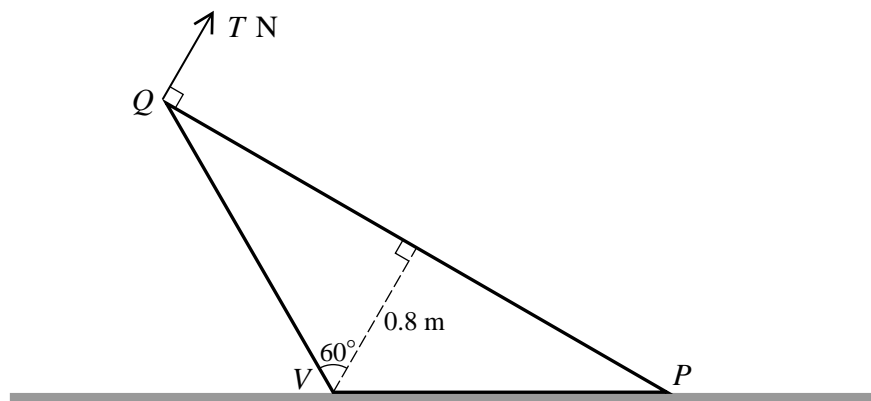**(i)** Calculate the least possible value of F. **[4]**

Fig. 2

The force of magnitude F N is removed and an increasing force of magnitude T N acting upwards in the vertical plane of symmetry of the cone and perpendicular to PQ is applied to the cone at Q (see Fig. 2). The coefficient of friction between the cone and the horizontal plane is μ.

(ii) Given that the cone slides before it topples about P, calculate the greatest possible value for μ.

[10]

(Q7, June 2011)

16
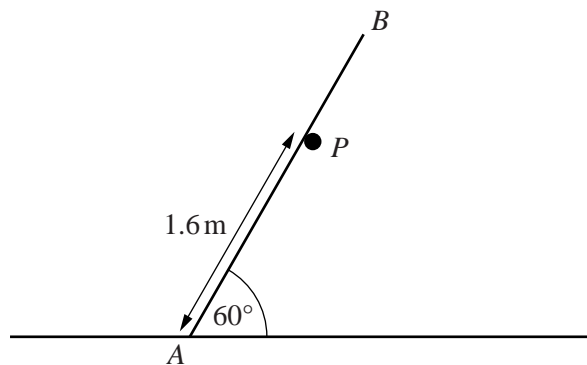
A uniform rod *AB* of mass 10 kg and length 2.4 m rests with *A* on rough horizontal ground. The rod makes an angle of 60° with the horizontal and is supported by a fixed smooth peg *P*. The distance *AP* is 1.6 m (see diagram).

  (i) Calculate the magnitude of the force exerted by the peg on the rod.   **[3]**

 (ii) Find the least value of the coefficient of friction between the rod and the ground needed to maintain equilibrium.   **[5]**

(Q3, Jan 2012)

17
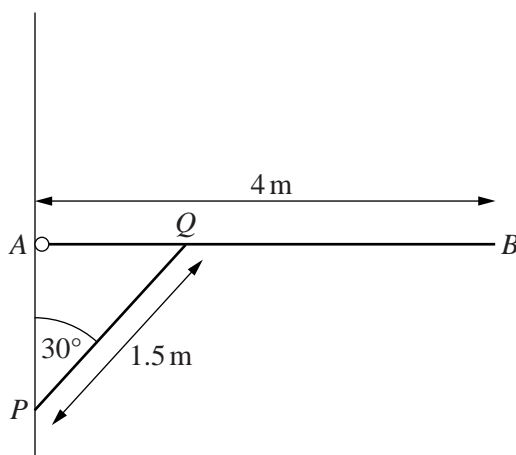
A uniform beam *AB* of mass 15 kg and length 4 m is freely hinged to a vertical wall at *A*. The beam is held in equilibrium in a horizontal position by a light rod *PQ* of length 1.5 m. *P* is fixed to the wall vertically below *A* and *PQ* makes an angle of 30° with the vertical (see diagram). The force exerted on the beam at *Q* by the rod is in the direction *PQ*. Find

  (i) the magnitude of the force exerted on the beam at *Q*,   **[3]**

 (ii) the magnitude and direction of the force exerted on the beam at *A*.   **[6]**

(Q3, June 2012)

18
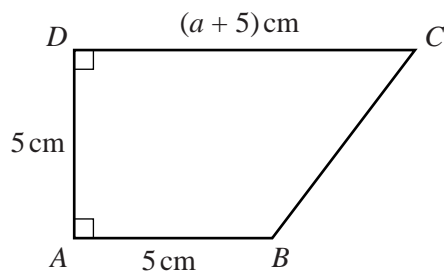
The diagram shows the cross-section through the centre of mass of a uniform solid prism. The cross-section is a trapezium *ABCD* with *AB* and *CD* perpendicular to *AD*. The lengths of *AB* and *AD* are each 5 cm and the length of *CD* is $(a + 5)$ cm.

the distance of the centre of mass of the prism from *AD* is $\dfrac{a^2 + 15a + 75}{3(a + 10)}$ cm.

The prism is placed with the face containing *AB* in contact with a horizontal surface.

(ii) Find the greatest value of *a* for which the prism does not topple. **[3]**

The prism is now placed on an inclined plane which makes an angle $\theta°$ with the horizontal. *AB* lies along a line of greatest slope with *B* higher than *A*.

(iii) Using the value for *a* found in part **(ii)**, and assuming the prism does not slip down the plane, find the greatest value of θ for which the prism does not topple. **[6]**

(Q7, June 2012)

physicsandmathstutor.com

19
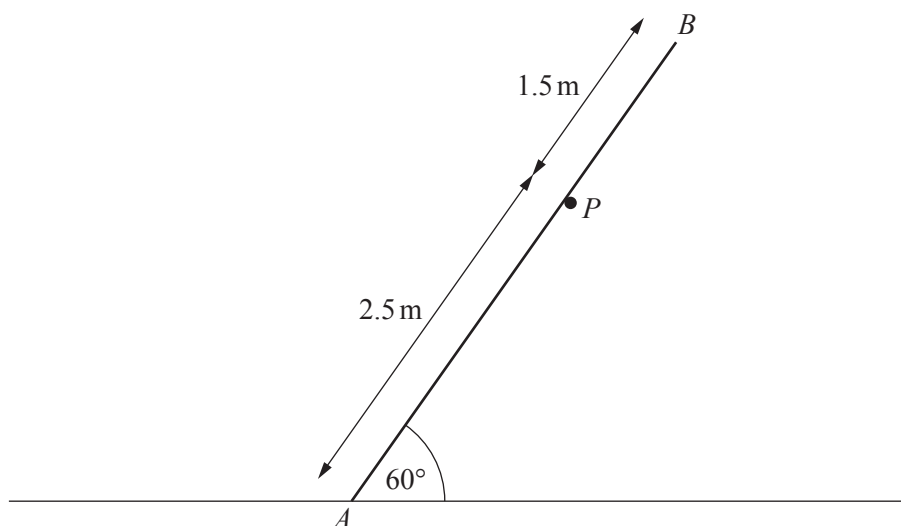
A uniform rod *AB*, of mass 3 kg and length 4 m, is in limiting equilibrium with *A* on rough horizontal ground. The rod is at an angle of 60° to the horizontal and is supported by a small smooth peg *P*, such that the distance *AP* is 2.5 m (see diagram). Find

(i) the force acting on the rod at *P*, **[3]**

(ii) the coefficient of friction between the ground and the rod. **[5]**

(Q5, Jan 2013)
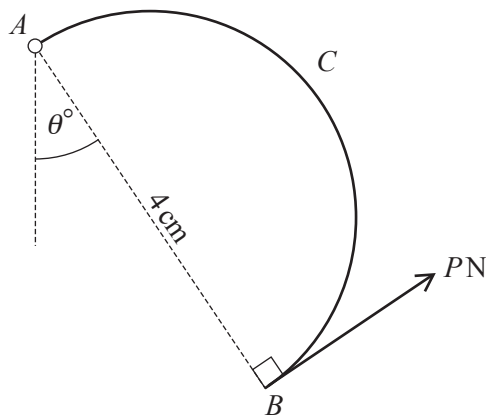
A uniform semicircular arc ACB is freely pivoted at A. The arc has mass $0.3\,\text{kg}$ and is held in equilibrium by a force of magnitude $P\,\text{N}$ applied at B. The line of action of this force lies in the same plane as the arc, and is perpendicular to AB. The diameter AB has length $4\,\text{cm}$ and makes an angle of $\theta°$ with the downward vertical (see diagram).

(i) Given that $\theta = 0$, find the magnitude of the force acting on the arc at A. **[6]**

(ii) Given instead that $\theta = 30$, find the value of P. **[4]**

(Q3, June 2013)

21 A solid uniform cone has height $8\,\text{cm}$, base radius $5\,\text{cm}$ and mass $4\,\text{kg}$. A uniform conical shell has height $10\,\text{cm}$, base radius $5\,\text{cm}$ and mass $0.4\,\text{kg}$. The two shapes are joined together so that the circumferences of their circular bases coincide.

(i) Find the distance of the centre of mass of the shape from the common circular base. **[4]**
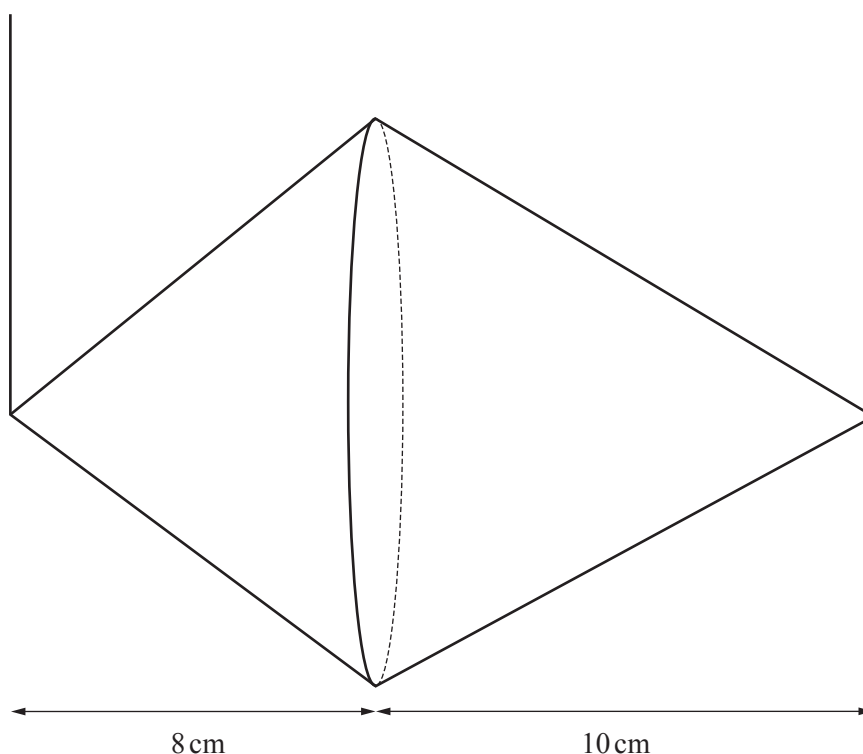
The object is suspended with a string attached to the vertex of the cone and another string attached to the vertex of the conical shell. The object is in equilibrium with the strings vertical and the axis of symmetry of the object horizontal (see diagram).

(ii) Find the tension in each string. **[4]**

(Q4, June 2013)